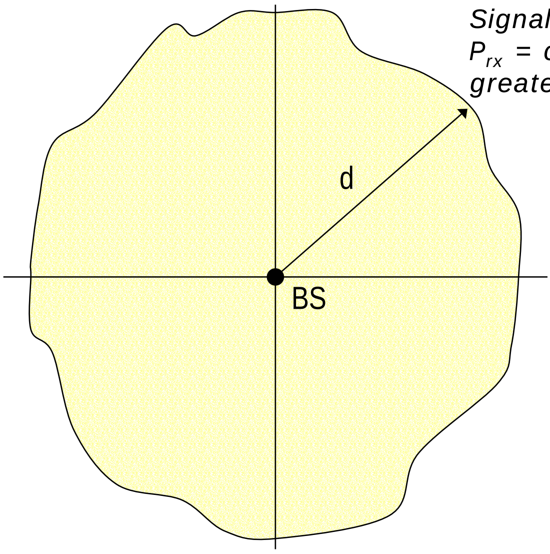
PART 2
Cellular Coverage Concepts

Lecture 2.1
why cells

Coverage for a terrestrial zone



Signal OK if P_{rx} > -X dBm
$P_{rx} = c \, P_{tx} \, d^{-4}$
greater P_{tx} → greater d

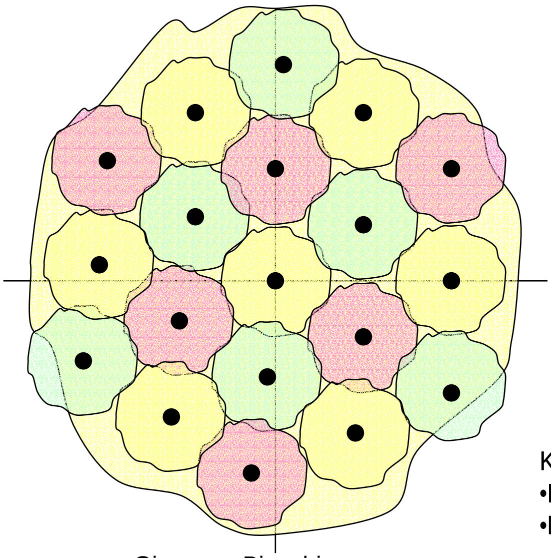
1 Base Station
N=12 channels
•(e.g. 1 channel = 1 frequency)

⬇

N=12 simultaneous calls

Cellular coverage

target: cover the same area with a larger number of BSs



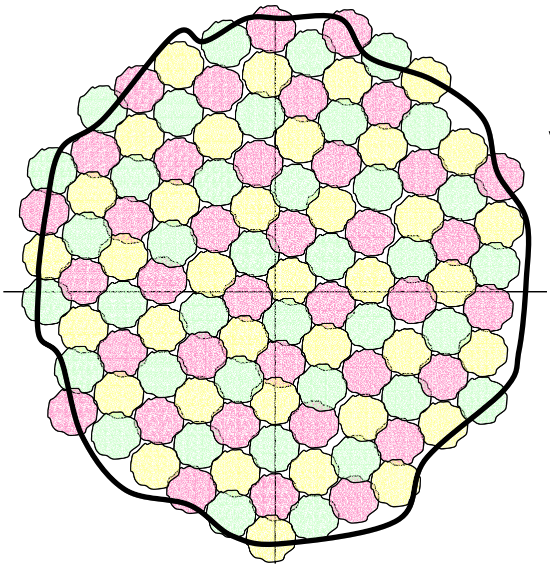
19 Base Station
12 frequencies
4 frequencies/cell

⬇

Worst case:
4 calls (all users in same cell)
Best case:
76 calls (4 users per cell)
Average case >> 12
Low transmit power

Key advantages:
•Increased capacity (freq. reuse)
•Decreased tx power

Cellular coverage (microcells)



many BS

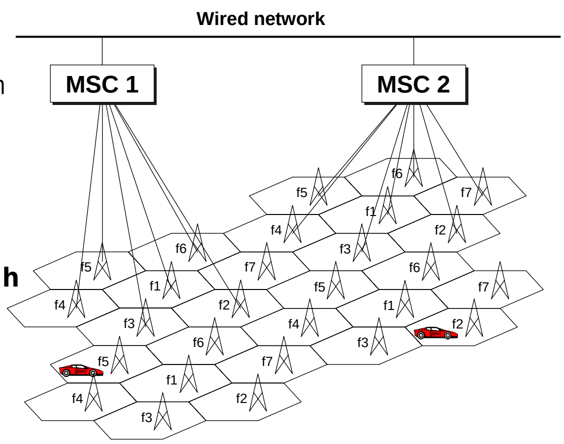
Very low power!!
Unlimited capacity!!

Usage of same spectrum
(12 frequencies)
(4 freq/cell)

Disadvantage:
mobility management

Cellular system architecture

- ➔ **1 BS per cell**
  - ⇨ Cell: Portion of territory covered by one radio station
  - ⇨ One or more carriers (frequencies; channels) per cell
- ➔ **Mobile users full-duplex connected with BS**
- ➔ **1 MSC controls many BSs**
- ➔ **MSC connected to PSTN**

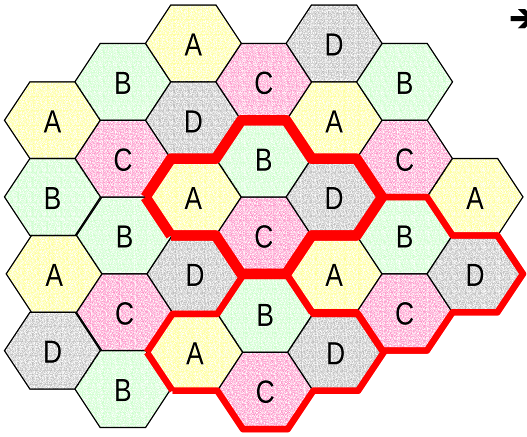


BS = Base Station
MSC = Mobile Switching Centre
PSTN = Public Switching Telephone Network

Cellular capacity

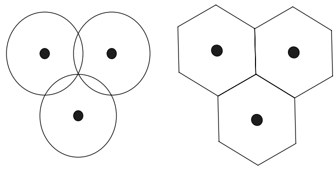
- ➔ **Increased via frequency reuse**
  - ⇨ Frequency reuse depends on interference
  - ⇨ need to sufficiently separate cells
    - →reuse pattern = cluster size (7 → 4 → 3): discussed later
- ➔ **Cellular system capacity: depends on**
  - ⇨ overall number of frequencies
    - →Larger spectrum occupation
  - ⇨ frequency reuse pattern
  - ⇨ Cell size
    - →Smaller cell (cell → microcell → picocell) = greater capacity
    - →Smaller cell = lower transmission power
    - →Smaller cell = increased handover management burden

hexagonal cells



➔ **Hexagon:**

⇨ Good approximation for circle



⇨ Ideal coverage pattern

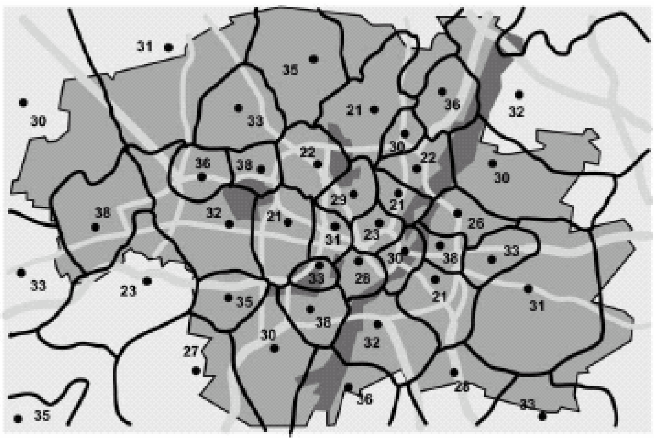
→no “holes”

→no cell superposition

➔**Example case:**

⇨Reuse pattern = 4

Cells in real world



Shaped by terrain, shadowing, etc

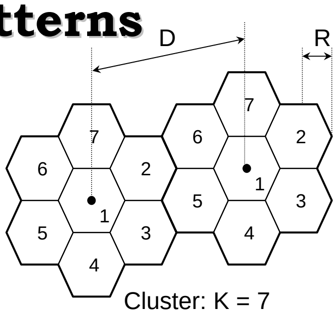
Cell border: local threshold, beyond which neighboring BS signal is received stronger than current one

PART 2
Cellular Coverage Concepts

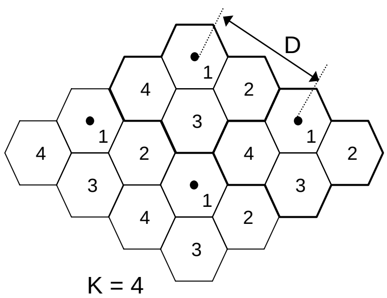
Lecture 2.2
Clusters and CCI

Reuse patterns

- ➔ **Reuse distance:**
  - ⇨ Key concept
  - ⇨ In the real world depends on
    - → Territorial patterns (hills, etc)
    - → Transmitted power
      - » and other propagation issues such as antenna directivity, height of transmission antenna, etc
- ➔ **Simplified hexagonal cells model:**
  - ⇨ reuse distance depends on reuse pattern (cluster size)
  - ⇨ Possible clusters:
    - → 3,4,7,9,12,13,16,19,…



Cluster: K = 7



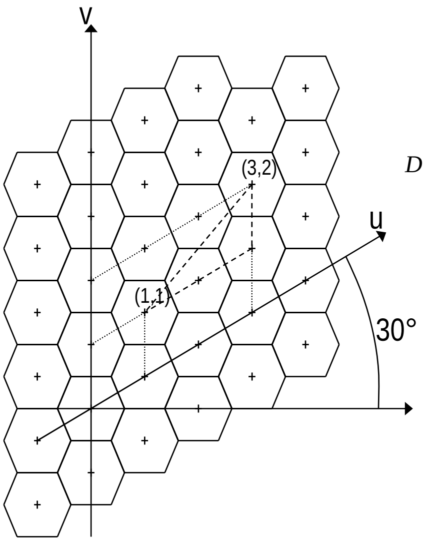
K = 4

Reuse distance

- ➔ **General formula** $\boxed{D = R\sqrt{3K}}$
- ➔ **Valid for hexagonal geometry**
- ➔ **D = reuse distance**
- ➔ **R = cell radius**
- ➔ **q = D/R =frequency reuse factor**

K	q=D/R
3	3,00
4	3,46
7	4,58
9	5,20
12	6,00
13	6,24

Proof



- ➔ **Distance between two cell centers:**
  - ⇨ $(u_1,v_1) \leftrightarrow (u_2,v_2)$

$$D = \sqrt{\left[(u_2 - u_1)\cos 30^o\right]^2 + \left[(v_2 - v_1) + (u_2 - u_1)\sin 30^o\right]^2}$$

  - ⇨ Simplifies to:

$$D = \sqrt{(u_2 - u_1)^2 + (v_2 - v_1)^2 + (u_2 - u_1)(v_2 - v_1)}$$

  - ⇨ Distance of cell (i,j) from (0,0):

$$D = \sqrt{i^2 + j^2 + ij}\,\sqrt{3}R$$

$$D_R = \sqrt{i^2 + j^2 + ij}$$

  - ⇨ Cluster: easy to see that

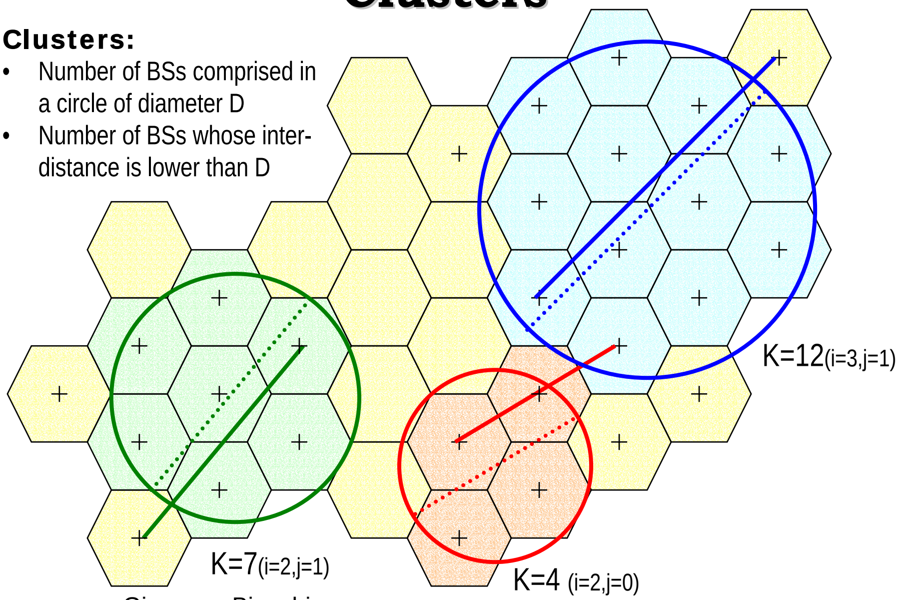
$$K = D_R^2 = i^2 + j^2 + ij$$

  - ⇨ hence: $D = R\sqrt{3K}$

Clusters

Clusters:

- Number of BSs comprised in a circle of diameter D
- Number of BSs whose inter-distance is lower than D

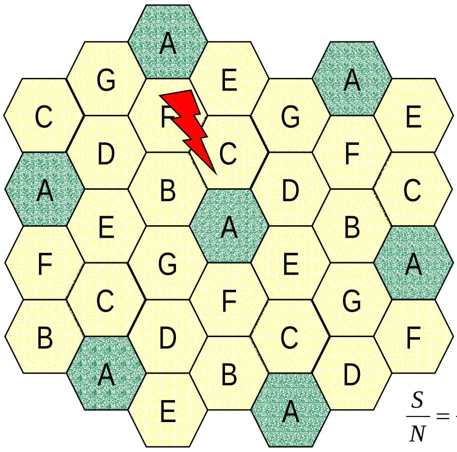


Possible clusters

all integer i,j values

i	j	K=ii+jj+ij	q=D/R
1	0	1	1,73
1	1	3	3,00
2	0	4	3,46
2	1	7	4,58
2	2	12	6,00
3	0	9	5,20
3	1	13	6,24
3	2	19	7,55
3	3	27	9,00
4	0	16	6,93
4	1	21	7,94
4	2	28	9,17
4	3	37	10,54
4	4	48	12,00
5	0	25	8,66
5	1	31	9,64

Co-Channel Interference



- **Frequency reuse implies that remote cells interfere with tagged one**
- **Co-Channel Interference (CCI)**
  - sum of interference from remote cells

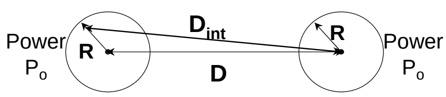
$$\frac{S}{N} = \frac{\text{signal power (S)}}{\text{noise power } (N_S) + \text{interfering signal power (I)}}$$

$$\frac{S}{I} = \frac{\text{signal power (S)}}{\text{interfering signal power (I)}}$$

$$\frac{S}{N} \approx \frac{S}{I} \quad \text{as } N_S \text{ small}$$

CCI Computation - assumptions

- **Assumptions**
  - N_I=6 interfering cells
    - N_I=6: first ring interferers only
    - we neglect second-ring interferers
  - Negligible Noise N_S
    - S/N ~ S/I
  - $d^{-\eta}$ propagation law
    - η=4 (in general)
  - Same parameters for all BSs
    - Same P_{tx}, antenna gains, etc
- **Key simplification**
  - Signal for MS at distance R
  - Signal from BS interferers at distance D



D_{int} ~ D

CCI computation

$$\frac{S}{N} \approx \frac{S}{I} = \frac{\text{cost} \cdot R^{-\eta}}{\sum_{k=1}^{N_I} \text{cost} \cdot D^{-\eta}} =$$

By using the assumptions of same cost and same D:

$$= \frac{1}{N_I}\left(\frac{R}{D}\right)^{-\eta} = \frac{1}{N_I}\left(\frac{D}{R}\right)^{\eta} = \frac{1}{N_I} q^{\eta}$$

Results depend on ratio q=D/R (q=frequency reuse factor)

Alternative expression: recalling that $D = R\sqrt{3K}$

$$\frac{S}{N} \approx \frac{S}{I} = \frac{1}{N_I}\left(\frac{R}{R\sqrt{3K}}\right)^{-\eta} = \frac{1}{N_I}(3K)^{\eta/2} = \frac{(3K)^{\eta/2}}{6}$$

N_I=6, μ=4 → $\frac{S}{I} = \frac{(3K)^2}{6} = \frac{3}{2}K^2$

USAGE: Given an S/I target, cluster size K is obtained

Examples

➔ **target conditions:**
- ⇨ S/I=9 dB
- ⇨ η=4

➔ **Solution:**

$$\frac{S}{I} = 10^{0.9} = 7.94 \approx 8$$

$$\frac{S}{I} = \left.\frac{(3K)^{\eta/2}}{6}\right|_{\eta=4} \Rightarrow K = \sqrt{\frac{2}{3}\cdot\frac{S}{I}}$$

$$K \geq 2.3 \quad \Rightarrow \quad K = 3$$

➔ **target conditions:**
- ⇨ S= 18dB
- ⇨ η=4.2

➔ **Solution:**

$$\frac{S}{I}[dB] = 5\eta\log(3K) - 10\log 6$$

$$\log(3K) = \frac{18+7.78}{21} = 1.23$$

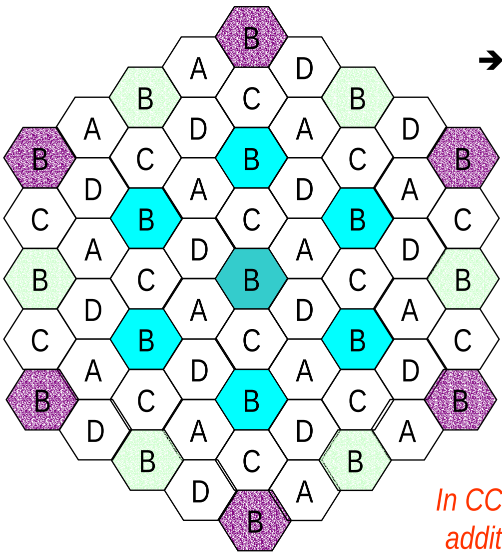
$$K \geq \frac{10^{1.23}}{3} = 5.63 \quad \Rightarrow \quad K = 7$$

S/I computation

assuming 6 interferers only (first ring)

K	q=D/R	S/I	S/I dB
3	3,00	13,5	11,3
4	3,46	24,0	13,8
7	4,58	73,5	18,7
9	5,20	121,5	20,8
12	6,00	216,0	23,3
13	6,24	253,5	24,0
16	6,93	384,0	25,8
19	7,55	541,5	27,3
21	7,94	661,5	28,2
25	8,66	937,5	29,7

Additional interferers

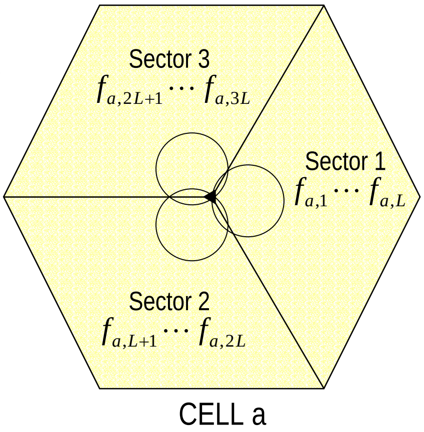


- ➔ **case K=4**
  - ⇨ note that for each cluster there are always N_I=6 first-ring interferers

In CCI computation, contribute of additional interferers is marginal

sectorization

- ➔ **Directional antennas**
- ➔ **Cell divided into sectors**
- ➔ **Each sector uses different frequencies**
  - ⇨ To avoid interference at sector borders
- ➔ **PROS:**
  - ⇨ CCI reduction
- ➔ **CONS:**
  - ⇨ Increased handover rate
  - ⇨ Less effective "trunking" leads to performnce impairments



CCI reduction via sectorization

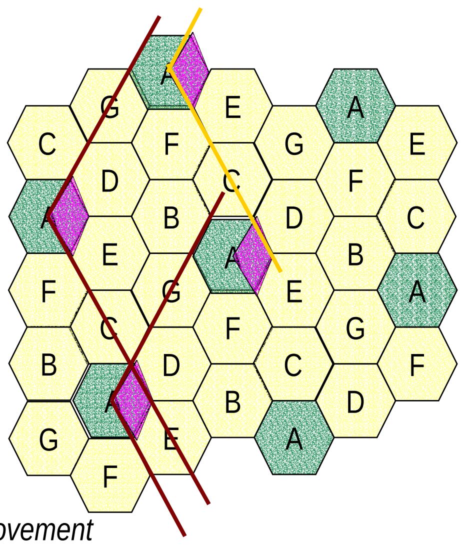
three sectors case

- ➔ **Inferference from 2 cells, only**
  - ⇨ Instead of 6 cells

With usual approxs (specifically, $D_{int} \sim D$)

$$\left[\frac{S}{I}\right]_{120^o} = \frac{R^{-\eta}}{2D^{-\eta}} = 3 \cdot \left[\frac{S}{I}\right]_{omni}$$

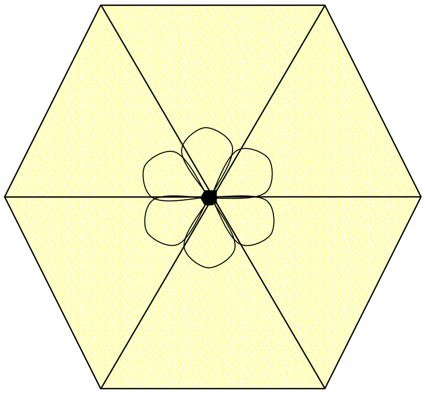
$$\left[\frac{S}{I}\right]_{120^o} dB = \left[\frac{S}{I}\right]_{omni} dB + 4.77$$



Conclusion: 3 sectors = 4.77 dB improvement

6 sectors

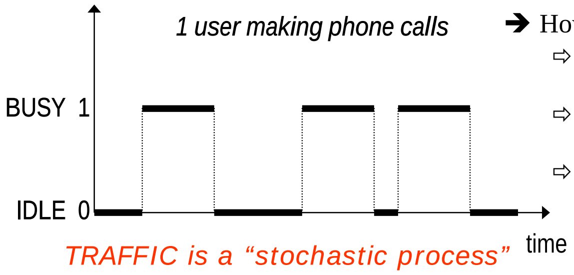
- ➔ 60º Directional antennas
- ➔ CCI reduction:
  - ⇨ 1 interfereer only
  - ⇨ 6 x S/I in the omni case
  - ⇨ Improvement: 7.78 dB



PART 2
Cellular Coverage Concepts

Lecture 2.3
teletraffic considerations,
teletraffic planning

Traffic generated by one user
(statistical notion of traffic)



TRAFFIC is a "stochastic process"

- ➔ How to characterize this process?
  - ⇨ statistical distribution of the "BUSY" period
  - ⇨ statistical distribution of the "IDLE" period
  - ⇨ statistical characterization of the process "memory"
    - → E.g. at a given time, does the probability that a user starts a call result different depending on what happened in the past?

➔ Traffic characterization suitable for traffic engineering

$$\text{traffic intensity } A_i = \lim_{\Delta t \to \infty} \frac{\text{amount of busy time in } \Delta t}{\Delta t} =$$
$$= (\text{average number } \lambda \text{ of calls per hour}) \times (\text{average call duration } \tau) =$$
$$= \text{probability that, at a random time t, user is in BUSY state} =$$
$$= \text{mean process value}$$

All equivalent (if stationary process)

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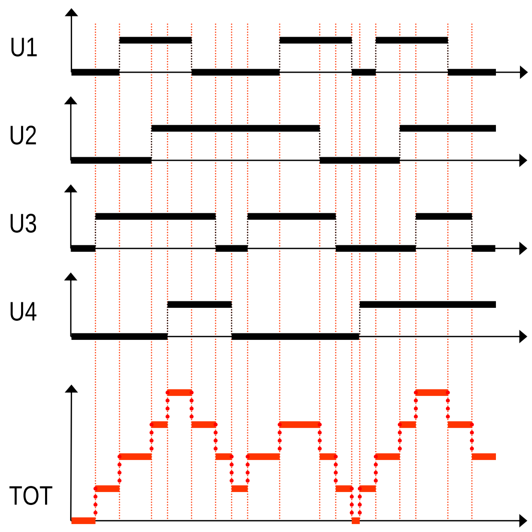
Traffic generated by more than one users



Traffic intensity
(adimensional, measured in Erlangs):

$$A = \sum_{i=1}^{4} A_i = 4A_i$$

$$P[\text{k active calls}] = \binom{4}{k} A_i^k (1 - A_i)^{4-k}$$
$$E[\text{active calls}] = 4 \cdot A_i = A$$

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example

- ➔ **5 users**
- ➔ **Each user makes an average of 3 calls per hour**
- ➔ **Each call, in average, lasts for 4 minutes**

$$A_i = 3\left[\frac{calls}{hour}\right]\times\frac{4}{60}[hours]=\frac{1}{5}[erl]$$

$$A = 5\times\frac{1}{5}[erl]=1[erl]$$

Meaning: in average, there is 1 active call; but the actual number of active calls varies from 0 (no active user) to 5 (all users active), with given probability

number of active users	probability
0	0,327680
1	0,409600
2	0,204800
3	0,051200
4	0,006400
5	0,000320

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Second example

- ➔ **30 users**
- ➔ **Each user makes an average of 1 calls per hour**
- ➔ **Each call, in average, lasts for 4 minutes**

$$A = 30\times\left(1\cdot\frac{4}{60}\right)=2 \text{ Erlangs}$$

SOME NOTES:
-In average, 2 active calls (intensity A);
-Frequently, we find up to 4 or 5 calls;
-Prob(n.calls>8) = 0.01%
-More than 11 calls only once over 1M

TRAFFIC ENGINEERING: how many channels to reserve for these users!

n. active users	binom	probab	cumulat
0	1	1,3E-01	0,126213
1	30	2,7E-01	0,396669
2	435	2,8E-01	0,676784
3	4060	1,9E-01	0,863527
4	27405	9,0E-02	0,953564
5	142506	3,3E-02	0,987006
6	593775	1,0E-02	0,996960
7	2035800	2,4E-03	0,999397
8	5852925	5,0E-04	0,999898
9	14307150	8,7E-05	0,999985
10	30045015	1,3E-05	0,999998
11	54627300	1,7E-06	1,000000
12	86493225	1,9E-07	1,000000
13	119759850	1,9E-08	1,000000
14	145422675	1,7E-09	1,000000
15	155117520	1,3E-10	1,000000
16	145422675	8,4E-12	1,000000
17	119759850	5,0E-13	1,000000
18	86493225	2,6E-14	1,000000
19	54627300	1,2E-15	1,000000
20	30045015	4,5E-17	1,000000
21	14307150	1,5E-18	1,000000
22	5852925	4,5E-20	1,000000
23	2035800	1,1E-21	1,000000
24	593775	2,3E-23	1,000000
25	142506	4,0E-25	1,000000
26	27405	5,5E-27	1,000000
27	4060	5,8E-29	1,000000
28	435	4,4E-31	1,000000
29	30	2,2E-33	1,000000
30	1	5,2E-36	1,000000

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A note on binomial coefficient computation

$$\binom{60}{12} = \frac{60!}{12!48!} = 1.39936e+12$$

but $60! = 8.32099e+81$ ***(overflow problems!!)***

$$\binom{60}{12} = \exp\left(\log\binom{60}{12}\right) = \exp(\log(60!) - \log(12!) - \log(48!)) =$$

$$= \exp\left(\sum_{i=1}^{60}\log(i) - \sum_{i=1}^{12}\log(i) - \sum_{i=1}^{48}\log(i)\right)$$ ***(no overflow!! before exp...)***

$$\binom{60}{12} A_i^{12}(1-A_i)^{48} =$$

$$= \exp\left(\sum_{i=1}^{60}\log(i) - \sum_{i=1}^{12}\log(i) - \sum_{i=1}^{48}\log(i) + 12\log(A_i) + 48\log(1-A_i)\right)$$

(no overflow!! never!)

Infinite Users

Assume M users, generating an overall traffic intensity A
(i.e. each user generates traffic at intensity A_i =A/M).
We have just found that

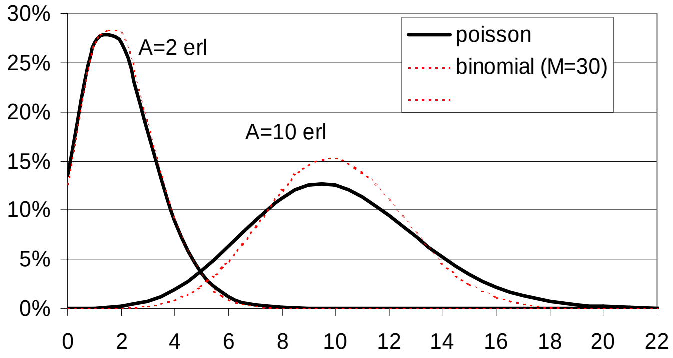
$$P[\text{k active calls, M users}] = \binom{M}{k} A_i^k (1-A_i)^{M-k} = \frac{M!}{(M-k)!k!}\left(\frac{A}{M}\right)^k \frac{\left(1-\frac{A}{M}\right)^M}{\left(1-\frac{A}{M}\right)^k}$$

Let M→infinity, while maintaining the same overall traffic intensity A

$$P[\text{k active calls}, \infty \text{ users}] = \lim_{M\to\infty} \frac{M!}{(M-k)!}\cdot\frac{1}{k!}\cdot\frac{A^k}{M^k}\cdot\left(1-\frac{A}{M}\right)^M \cdot\left(1-\frac{A}{M}\right)^{-k} =$$

$$= \frac{A^k}{k!}\cdot\lim_{M\to\infty}\frac{M(M-1)\cdots(M-k+1)}{M^k}\cdot\left[\left(1-\frac{A}{M}\right)^{-\frac{M}{A}}\right]^{-A}\cdot\left(1-\frac{A}{M}\right)^{-k} = e^{-A}\frac{A^k}{k!}$$

Poisson Distribution



$$P_k(A) = e^{-A}\frac{A^k}{k!}$$

Very good matching with Binomial
(when M large with respect to A)

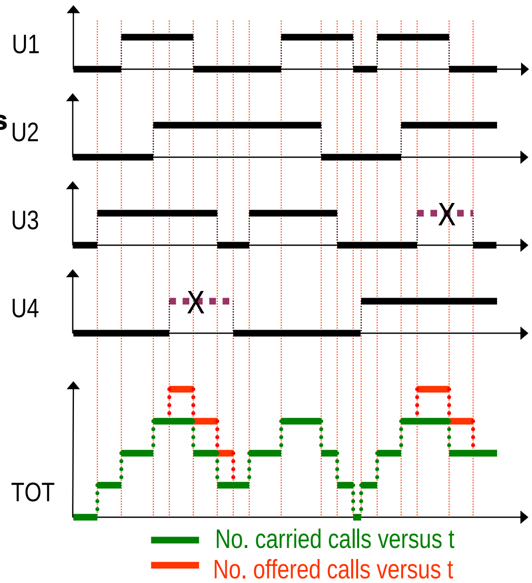
Much simpler to use than Binomial
(no annoying queueing theory complications)

Limited number of channels

THE most important problem
in circuit switching

- ➔ **The number of channels C is less than the number of users M (eventually infinite)**
- ➔ **Some offered calls will be "blocked"**
- ➔ **What is the blocking probability?**
  - ⇨ We <u>have</u> an expression for P[k offered calls]
  - ⇨ We <u>must find</u> an expression for P[k accepted calls]
  - ⇨ As:

$$P[\text{block}] = P[\text{C accepted calls}]$$

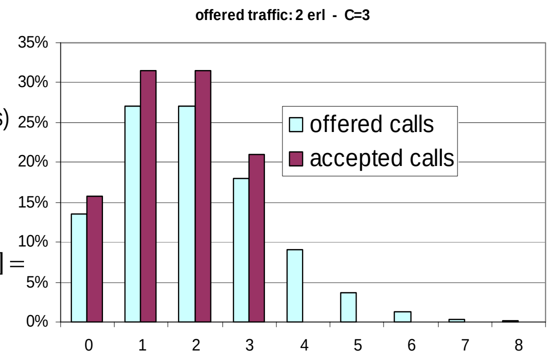


Channel utilization probability

- C channels available
- Assumptions:
  - Poisson distribution (infin. users)
  - Blocked calls cleared
- It can be proven (from Queueing theory) that:

$$P[\text{k calls in the system, k} \in (0,C)] = \frac{P[\text{k offered calls}]}{\sum_{i=0}^{C} P[\text{i offered calls}]}$$

(very simple result!)



- Hence: $$P[\text{system full}] = P[\text{C accepted calls}] = \frac{P[\text{C offered calls}]}{\sum_{i=0}^{C} P[\text{i offered calls}]}$$

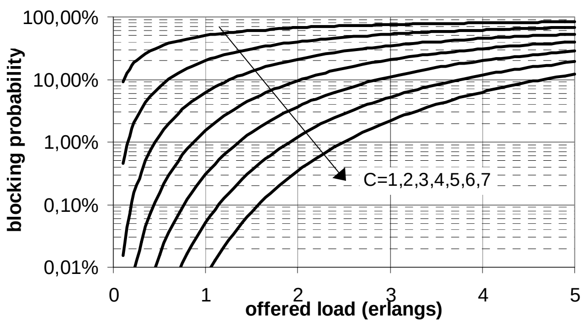
Blocking probability: Erlang-B

- Fundamental formula for telephone networks planning
  - A_o=offered traffic in Erlangs

$$\Pi_{block} = \frac{\frac{A_o^C}{C!}}{\sum_{j=0}^{C} \frac{A_o^j}{j!}} = E_{1,C}(A_o)$$

- Efficient recursive computation available

$$E_{1,C}(A_o) = \frac{A_o E_{1,C-1}(A_o)}{C + A_o E_{1,C-1}(A_o)}$$



NOTE: finite users

- Erlang-B obtained for the infinite users case
- It is easy (from queueing theory) to obtain an explicit blocking formula for the finite users case:
- ENGSET FORMULA:

$$\Pi_{block} = \frac{A_i^C \binom{M-1}{C}}{\sum_{k=0}^{C} A_i^k \binom{M-1}{i}}$$

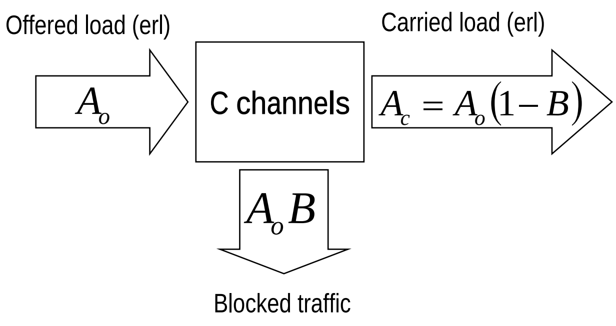
$$A_i = \frac{A_o}{M}$$

- Erlang-B can be re-obtained as limit case
  - $M \rightarrow$ infinity
  - $A_i \rightarrow 0$
  - $M \cdot A_i \rightarrow A_o$
- Erlang-B is a very good approximation as long as:
  - A/M small (e.g. <0.2)
- In any case, Erlang-B is a conservative formula
  - yields higher blocking probability
  - Good feature for planning

Capacity planning

- **Target: support users with a given Grade Of Service (GOS)**
  - GOS expressed in terms of upper-bound for the blocking probability
    - GOS example: subscribers should find a line available in the 99% of the cases, i.e. they should be blocked in no more than 1% of the attempts
- **Given:**
    - C channels
    - Offered load A_o
    - Target GOS B_{target}
  - C obtained from numerical inversion of $B_{\text{target}} = E_{1,C}(A_o)$

Channel usage efficiency



efficiency: $\eta = \frac{A_c}{C} = \frac{A_o\left(1-E_{1,C}(A_o)\right)}{C} \approx \frac{A_o}{C}$ if small blocking

Fundamental property: for same GOS, efficiency increases as C grows!!

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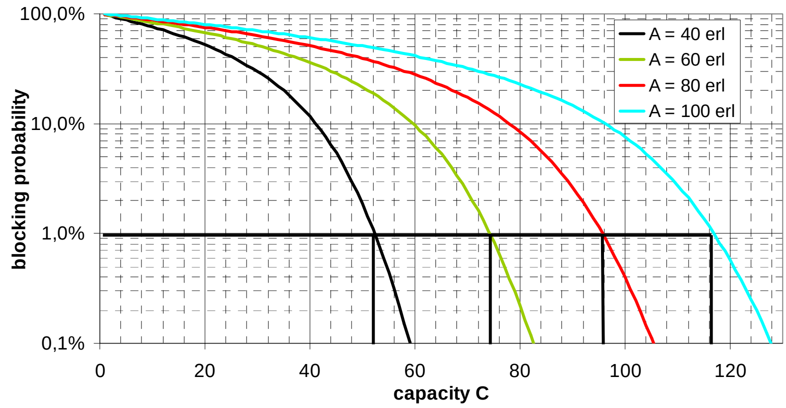
example



GOS = 1% maximum blocking.
Resulting system dimensioning
and efficiency:

40 erl	C >= 53	η = 74.9%
60 erl	C >= 75	η = 79.3%
80 erl	C >= 96	η = 82.6%
100 erl	C >= 117	η = 84.6%

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Erlang B calculation - tables

Example: How many channels are required to support 100 users with a GOS of 2% if the average traffic per user is 30 mE?

100x30mE = 3 Erlangs
3 Erlangs @ 2% GOS =

8 channels

Trunks	0.01	0.015	0.02	0.03
P(B)=				
1	0.010	0.015	0.020	0.031
2	0.153	0.190	0.223	0.282
3	0.455	0.536	0.603	0.715
4	0.870	0.992	1.092	1.259
5	1.361	1.524	1.657	1.877
6	1.913	2.114	2.277	2.544
7	2.503	2.743	2.936	3.250
8	3.129	3.405	3.627	3.987
9	3.783	4.095	4.345	4.748
10	4.462	4.808	5.084	5.529

Erlang B calculation - software

- ➔**Erlang-B formula very easy to implement**
  - ⇨Even if some tricks needed for numerical accuracy
- ➔**Erlang-B inversion not so easy**
  - ⇨Software tools
- ➔**Online calculator:**
  - ⇨http://mmc.et.tudelft.nl/~frits/Erlang.htm
  - ⇨Given two parameter, calculates the third
    - →N = number of circuits
    - →B = blocking probability
    - →A = offered load

Application to cellular networks

Cell size (radius R) may be determined on the basis of traffic considerations

- **First step:**
  - Given num channels and GOS
    - C=50 available channels in a cell
    - Blocking probability<=2%
  - Evaluate maximum cell (offered) load
    - From Erlang-B inversion(tables) A=40.25 erl
- **Second step**
  - Given traffic generated by each user
    - Each user: 4 calls/busy-hour
    - Each call: 2 min in average
    - A_i=4x2/60=0.1333 erl/user
  - Evaluate max num of users in cell
    - M=301.87 ~ 302
- **Third step:**
  - Given density of users
    - δ=500 users/km^2
  - Evaluate cell radius

$$\delta = \frac{M}{\pi R^2} \quad \Rightarrow \quad R = \sqrt{\frac{M}{\pi\delta}}$$

  - R~438m

Other example

- Three service providers are planning to provide cellular service for an urban area. The target GOS is 2% blocking. Users make 3 calls/busy-hour, each lasting 3 minutes in average (A_i=3/20=0.15)
  - Question: how many users can support each provider?
- Provider A configuration: 20 cells, each with 40 channels
- Provider B configuration: 30 cells, each with 30 channels
- Provider C configuration: 40 cells, each with 20 channels

- **Provider A:**
  - 40 channels/cell
  - at 2%: A_0=30.99 erl/cell
  - 619.8 erl-total
  - M=4132 overall users
- **Provider B:**
  - 30 channels/cell
  - at 2%: A_0=21.93 erl/cell
  - 654.9 erl-total
  - M=4386 overall users
- **Provider C:**
  - 20 channels/cell
  - at 2%: A_0=13.18 erl/cell
  - 527.2 erl-total
  - M=3515 overall users

Compare case A with C! The reason is the lower efficiency of 20 channels versus 40

Sectorization and traffic

- ➔ **Assume cluster K=7**
- ➔ **Omnidirectional antennas: CCI=18.7 dB**
- ➔ **120º sectors: CCI=23.4 dB**
- ➔ **60º sectors: CCI=26.4 dB**

- ➔ **Sectorization yields to better CCI**
- ➔ **BUT: the price to pay is a much lower trunking efficiency!**

- ➔ **With 60 channels/cell, GOS=1%,**
  - ⇨ Omni: 60 channels A_0=1x46.95= 46.95 erl η=77.5%
  - ⇨ 120º: 60/3=20 channels A_0=3x12.03= 36.09erl η=59.5%
  - ⇨ 60º: 60/6=10 channels A_0=6x4.46= 26.76erl η=44.1%

conclusion

- ➔ This module has given some hints regarding:
  - ⇨ Cell sizing via propagation considerations
  - ⇨ Frequency reuse via propagation considerations
  - ⇨ Cell planning via teletraffic consideration
- ➔ Very elementary models
  - ⇨ But sufficient to understand what's inside planning
- ➔ No mobility!
  - ⇨ Teletraffic models need to be extended to manage handover rates!
  - ⇨ Blocking requirement for an handover call MUST be much lower than blocking for a new incoming call
    - →severe math complications
    - →Guard channels for handover
    - →Out of the scopes of this class!